Time Series Analysis

Figure 2.1 Non-stationary series $y_t = y_{t-1} + u_t$, $u_t \sim IN(0, 1)$

Source: Charemza and Deadman (1992)

$y_t = y_{t-1} + 0.1 + u_t$; $x_t = 0.1t + u_t$ $u_t \sim IN(0, 1)$

Figure 2.3 Non-stationary series with drift, y_t, and trend-stationary series, x_t

Source: Charemza and Deadman (1992)

Critical Values for the ADF Test.

ADF Variant	Significance level	Critical values		
		n = 25	n = 50	n = 100
No constant, no deterministic trend	0.05	-1.95	-1.95	-1.95
	0.01	-2.66	-2.62	-2.60
Constant, no deterministic trend	0.05	-3.00	-2.93	-2.89
	0.01	-3.75	-3.58	-3.51
Constant and deterministic trend	0.05	-3.60	-3.50	-3.45
	0.01	-4.38	-4.15	-4.04

Source: Fuller (1976). Note: Many software packages now report appropriate critical values for the ADF statistic. These are usually based upon calculations undertaken by MacKinnon (1991, 1996).

ADF Test Results.

	Single unit root[a]	Second unit root[a]
Bilateral exchange rate (JPY/GBP)*	-3.043 (1)	-6.555 (0) **
UK wholesale prices*	-3.233 (1)	-5.182 (0) **
Japanese wholesale prices*	-1.929 (1)	-3.922 (0) *
UK short term interest rate	-2.113 (0)	-8.745 (0) **
Japanese short term interest rate	-3.229 (1)	-5.174 (0) **

Quarterly data 1974Q1 to 1995Q4.
* Natural logarithm.
[a] All ADF tests include a constant and a deterministic trend. Figures in parentheses indicate the number of lags required to obtain residuals free from autocorrelation. Critical values at the 5% and 1% levels are -3.45 and -4.04 respectively.
* Significant at the 5% level. ** Significant at the 1% level.

Engel-Granger 2 Step Procedure

Estimate the long run relationship using ordinary least squares regression. Test for cointegration using the Augmented Dickey-Fuller test statistic. If the residuals from the long run equation are stationary, estimate the short run (error correction) relationship using ordinary least squares regression.

Long Run Cointegrating Results.
Dependent Variable: JPY/GBP Exchange Rate*.

Regressors	Coefficient estimates
UK wholesale price index*	-0.9877
Japanese wholesale price index*	1.3725
UK short term interest rate	1.0519
Japanese short term interest rate	1.6147
Constant	3.4555
R^2	0.9781
CRDW	0.6329
ADF	-5.003 (1)[a]
Number of observations	88

Quarterly data 1974Q1 to 1995Q4.
* Natural logarithm.
[a] ADF test includes a constant but no deterministic trend. Figure in parentheses indicate the number of lags required to obtain residuals free from autocorrelation. Critical values at the 5% and 1% levels are -4.58 and -5.28 respectively.

Critical Values for Cointegration Tests.

Number of Variables	Significance level	Critical values		
		n = 50	n = 100	n = 200
2	0.05	-3.67	-3.37	-3.37
	0.01	-4.32	-4.07	-4.00
3	0.05	-4.11	-3.93	-3.78
	0.01	-4.84	-4.45	-4.35
4	0.05	-4.35	-4.22	-4.18
	0.01	-4.94	-4.75	-4.70
5	0.05	-4.76	-4.58	-4.48
	0.01	-5.41	-5.28	-5.02

Source: Engle and Yoo (1987). Note: Many software packages now report appropriate critical values for the ADF statistic. These are usually based upon calculations undertaken by MacKinnon (1991, 1996).

Dynamic Error Correction Model.
Dependent Variable: First Difference of JPY/GBP Exchange Rate*.

Variable	Coefficient	t statistic	Significance
First difference of UK wholesale price index*	-1.0549	-2.7541	0.007
First difference of Japanese wholesale price index*	2.6230	6.0980	0.000
First Difference of UK short term interest rate	-0.2717	-0.7766	0.440
First Difference of Japanese short term interest rate	-0.2528	-0.3504	0.727
Speed of adjustment	-0.3781	-5.2283	0.000
Constant	-0.0012	-0.1420	0.887
R-squared	0.4383		
Adjusted R-squared	0.4036		
S.E. of regression	0.0432		
F-statistic	12.6401	(0.000)	
Durbin-Watson	1.4785	(0.001, 0.028)	
Breusch-Godfrey serial correlation test	10.6703	(0.001)	
Ramsey's RESET test	1.6237	(0.206)	
LM heteroscedasticity test	12.9487	(0.000)	
Jarque-Bera test	0.6768	(0.713)	
Number of observations	87		

Estimation period 1974Q2 to 1995Q4.
* Natural logarithm.

Modelling Volatility

Most previous models volatility has been constant: $u_t \sim N(0, \sigma^2)$ or $u_t \sim N(0, 1)$.

Instead let variance of u_t change with time: $u_t \sim N(0, \sigma_t^2)$.

Autoregressive Conditional Heteroscedasticity (ARCH)

Mean Equation $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$

$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t$

Variance Equation $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + v_t$

$\sigma_t^2 = v_t (\alpha_0 + \alpha_1 u_{t-1}^2)$

$v_t \sim N(0, \sigma^2)$

Generalized Autoregressive Conditional Heteroscedasticity (GARCH)

Mean Equation $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$

$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t$

Variance Equation $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + v_t$

$\sigma_t^2 = v_t (\alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 \sigma_{t-1}^2)$

$v_t \sim N(0, \sigma^2)$

GARCH-in-Mean (GARCH-M)

Mean Equation $y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 x_{1t} + \beta_3 \sigma_t^2 + u_t$

$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 x_{1t} + \beta_3 \sigma_t + u_t$

Variance Equation $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + v_t$

$\sigma_t^2 = v_t (\alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 \sigma_{t-1}^2)$

$v_t \sim N(0, \sigma^2)$